

## Math 43 Midterm 3 Review

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

- [1] Consider the vector  $\vec{m} = \langle -2, -4 \rangle$ , and the vector  $\vec{n}$  with direction angle  $\frac{2\pi}{3}$  such that  $\|\vec{n}\| = 8$ .
- [a] Find a unit vector perpendicular to  $\vec{m}$ . (Do **NOT** use decimal approximations.)
  - [b] Find the direction angle of  $\vec{m}$ . (Your answer should be in radians, rounded to 2 decimal places.)
  - [c] Write  $2\vec{n} - \vec{m}$  as a linear combination of  $\vec{i}$  and  $\vec{j}$ . (Do **NOT** use decimal approximations.)
- [2] Consider the vectors  $\vec{f} = 2\vec{j} - 3\vec{k}$  and  $\vec{g} = -\vec{i} - 3\vec{j} + 4\vec{k}$ .
- [a] Find the angle between  $\vec{f}$  and  $\vec{g}$ . (Your answer should be in radians, rounded to 2 decimal places.)
  - [b] Find a unit vector perpendicular to both  $\vec{f}$  and  $\vec{g}$ . (Do **NOT** use decimal approximations.)
  - [c] Write  $\vec{f}$  as the sum of 2 vectors, one parallel to  $\vec{g}$  and one perpendicular to  $\vec{g}$ . (Do **NOT** use decimal approximations.)
  - [d] If the terminal point of  $\vec{g}$  is  $(-7, 4, -8)$ , find the initial point.
  - [e] If  $\vec{h} = a\vec{i} + b\vec{j} - 5\vec{k}$  is parallel to  $\vec{g}$ , find the values of  $a$  and  $b$ .
  - [f] If  $\vec{e} = 7\vec{i} + c\vec{j} - 5\vec{k}$  is perpendicular to  $\vec{g}$ , find the value of  $c$ .
- [3] Let  $P$  be the point  $(-5, -2, 3)$ . Let  $Q$  be the point  $(3, 2, -1)$ . Let  $R$  be the point  $(-3, 4, -2)$ .  
Let  $\vec{u}$  be the vector with initial point  $R$  and terminal point  $Q$ .  
Let  $\vec{w}$  be the vector with initial point  $P$  and terminal point  $R$ .
- [a] In which octant is  $R$ ?
  - [b] If you start at point  $P$ , move 2 units down, 4 units back, and 6 units to the right, find the co-ordinates of your ending point.
  - [c] Write  $\vec{u}$  in component form.
  - [d] Write  $\vec{w}$  as a linear combination of  $\vec{i}$ ,  $\vec{j}$  and  $\vec{k}$ .
  - [e] Find the magnitude of  $\vec{w}$ . (Do **NOT** use decimal approximations.)
  - [f] Find a unit vector in the opposite direction as  $\vec{w}$ . (Do **NOT** use decimal approximations.)
  - [g] Find a vector of magnitude 6 in the same direction as  $\vec{u}$ . (Do **NOT** use decimal approximations.)
  - [h] If  $\|\vec{v}\| = 3$ , and the angle between  $\vec{u}$  and  $\vec{v}$  is 2 radians, find  $\vec{u} \cdot \vec{v}$ . (Round your answer to 2 decimal places.)
  - [i] If  $\|\vec{v}\| = 3$ , and the angle between  $\vec{u}$  and  $\vec{v}$  is 2 radians, find the magnitude of  $\vec{u} \times \vec{v}$ .  
(Round your answer to 2 decimal places.)
  - [j] Find the area of triangle  $PQR$ . (Do **NOT** use decimal approximations.)
  - [k] Find  $\angle QRP$ . (Round your answer to 2 decimal places.)
  - [l] If a force represented by the vector  $4\vec{i} - 5\vec{k}$  is applied to an object as it moves from  $Q$  to  $P$ , find the work done.
  - [m] Find the general equation of the plane passing through  $P$ ,  $Q$  and  $R$ .
  - [n] Find parametric equations for the line which passes through  $P$  and is also parallel to  $\vec{u}$ .
  - [o] Find symmetric equations for the line which passes through  $Q$  and is also perpendicular to the plane  $-2x - 3y + z = 9$ .
  - [p] Find the equation of the sphere with  $P$  and  $Q$  as endpoints of a diameter.
- [4] Which octant or octants contain all points  $(x, y, z)$  where  $xz < 0$ ?
- [5] Consider the sphere  $x^2 + y^2 + z^2 - 4x + 6y + 10z + 29 = 0$ .
- [a] Find the center and radius of the sphere.
  - [a] Find the equations of the  $xy$ -,  $xz$ - and  $yz$ -traces of the sphere, and describe each trace.